Radial Oscillations of Hybrid Stars

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Abstract

We study the effect of quark and nuclear matter mixed phase on the radial oscillation modes of neutron stars. For this study we have considered recent models from two classes of equations of state namely the relativistic mean field theoretical models and models based on realistic nucleon-nucleon interactions incorporating relativistic corrections and three body nuclear interactions.

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1   Introduction

Recently there have been important developments in the determination of neutron star masses in binary systems. Where as the best determination of neutron star masses, all in the range 1.35 ± 0.04 $M_\odot$ (Thorsett 1999), is found for binary radio pulsars, limits on the masses of neutron stars from measurements of kilohertz quasiperiodic oscillation frequencies in low mass X-ray binaries lie in the region of $2M_\odot$ (Miller 1998a; 1998b; 1998c; see also van-der Klis 1998). In addition several X-ray binary masses have also been found to be high viz. Vela X-1 and Cygnus X-2 with masses $(1.8 \pm 0.2)M_\odot$ and $(1.8 \pm 0.4)M_\odot$ respectively (van Kerkwijk 1995; Orsoz 1999). These large masses put severe constraints on the equation of state(EOS) of dense matter and only very stiff EOS's are capable of sustaining such high masses. There are esentially two classes of EOS's for dense matter, one class based on relativistic mean field type of models with coupling constants treated as parameters to be fitted to observable quantities and the other based on realistic interactions between constituents obtained by fitting scattering data and using techniques of many body theory to evaluate energy and pressure. Both classes of models however, suffer from lack of sufficient experimental data. This situation is likely to be ameleorated by our understanding of the physics of heavy-ion collisions in laboratory experiments in near future. In the core of neutron stars where densities rise few times the normal nuclear densities, the state of matter is essentially unknown and the possibility of a phase transition to constituent quark matter or to hadronic matter containing hyperons and/or meson condensates has been extensively studied in the litrature (see for example Heiselberg 2001 and references therein). All these phase transitions whether first order or second result in softening of the EOS and thus aggravating the maximum mass limit. In a phase transition to quark matter, the neutron star may have a core of pure quark matter with a mantle of nuclear matter surrounding it and the two phases co-existing in a first order phase transition. Alternatively one may have a so called hybrid star discussed by Glendenning (Glendenning 1992; ibid 1997; see also Heiselberg 2001), wherein the quark and nuclear matter coexist in mixed phase with continuous pressure and density variation - a situation obtained by applying Gibb's criteria to a two-component system.

One important aspect of various compact stars is the study of their radial modes of oscillation. Radial oscillations give information about the stability of the stellar structure under consideration, and are thus quite important in distinguishing between various models of the stellar structure. Also as is well known, radial oscillations do not couple to gravitational waves; as a result the equations governing radial oscillations are quite simple and it is relatively easy to solve the eigenvalue problem that leads to the descrete set of eigenfrequencies for the system. The eigenfrequencies form a complete set and hence it is possible to describe any arbitrary periodic radial disturbance as a superposition of its various eigenmodes.

For neutron stars, beginning with Chandrasekhar (1964), radial modes have been investigated in the literature for more than thirty years and for various nuclear matter equations of state. (see for example Chanmugam 1977; Glass and Lindblom 1983; Vath and Chanmugam 1992; Kokkotas 2001) . In view of the fact that only a few EOS viz. those of Wiringa, Fiks and Fabrocini (Wiringa 1988) using A14+UV11 and U14+UV11 interactions and the recent EOS of Akaml, Pandharipande and Ravenhall (Akaml 1998) incorporating relativistic corrections and three- nucleon interactions in a new nucleon-nucleon interaction model (the Argonne A18 potential) are practically the only viable neutron star models capable of giving neutron star masses greater than 1.9 $M_{\odot}$, we have picked them for the present study. In addition we also consider the EOS given by Glendenning from the class of relativistic mean field theoretical models (Glendenning 1997). These equations invariably allow a phase transition in the core. In this paper we study the effect of a mixed quark-nuclear matter core on radial oscillations of hybrid stars.

2 Radial Oscillations of a non-rotating Star

The equations governing the radial oscillations of a non-rotating star, using static, spherically symmetric metric

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{1}$$

were given by Chandrasekhar (Chandrasekhar 1964). The structure of the star in hydrostatic equilibrium is determined by the Tolman-Openheimer-Volkoff equations

$$\frac{dp}{dr} = \frac{-G(p+\rho)(m+4\pi r^3 p)}{r^2(1-\frac{2GM}{r})} \tag{2}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho \tag{3}$$

$$\frac{d\nu}{dr} = \frac{2GM(1+\frac{4\pi r^3 p}{m})}{r(1-\frac{2GM}{r})} \tag{4}$$

where we have put $c = 1$. Assuming a radial displacement δr with harmonic time dependence $\delta r \sim e^{i\omega t}$ and defining variables $\xi = \frac{\delta r}{r}$ and $\zeta = r^2 e^{-\nu}\xi$, the equation governing radial adiabatic oscillations is given by

$$F\frac{d^2\zeta}{dr^2} + G\frac{d\zeta}{dr} + H\zeta = \omega^2\zeta \tag{5}$$

where

$$F = -\frac{e^{2\nu-2\lambda}(\gamma p)}{p+\rho} \tag{6}$$

$$G = -\frac{e^{2\nu-2\lambda}}{p+\rho}\left[\gamma p(\lambda+3\nu) + \frac{d(\gamma p)}{dr} - \frac{2}{r}(\gamma p)\right] \tag{7}$$

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$$H = \frac{e^{2\nu-2\lambda}}{p+\rho}\left[\frac{4}{r}\frac{dp}{dr} + 8\pi G e^{2\lambda}p(p+\rho) - \frac{1}{p+\rho}(\frac{dp}{dr})^2\right] \tag{8}$$

λ is related to the metric function through

$$e^{-2\lambda} = (1 - \frac{2GM(r)}{r}) \tag{9}$$

and γ is the adiabatic index, related to the speed of sound through

$$\gamma = \frac{p+\rho}{p}\frac{dp}{d\rho} \tag{10}$$

Equation(5) is solved under the boundary conditions

$$\zeta(r=0) = 0 \qquad \delta p(r=R) = 0 \tag{11}$$

where $\delta p(r)$ is given by

$$\delta p(r) = -\frac{dp}{dr}\frac{e^\nu\zeta}{r^2} - \frac{\gamma p e^\nu}{r^2}\frac{d\zeta}{dr} \tag{12}$$

Equation (5) with the boundary condition (11) represent a Sturm-Liouville eigenvalue problem for ω^2 with the well known result that the frequency spectrum is descrete. For $\omega^2 > 0$, ω is real and the solution is purely oscillatory whereas for $\omega^2 < 0$, ω is imaginery resulting in exponentially growing unstable radial oscillations. Another important consequence is that if the fundamental radial mode ω_0 is stable, so are the rest of the radial modes. For neutron stars ω_0 becomes imaginery at central densities $\rho_c > \rho_c^{critical}$ for which the star attains it's maximum mass. For $\rho_c = \rho_c^{critical}$, the fundamental frequency ω_0 vanishes and becomes unstable for higher densities and the star is no longer stable. There also exists another unstable point at the lower end of the central density, namely there exists a minimum mass for a stable neutron star and the frequency of the fundamental mode at the minimum mass again goes to zero.

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3 Results

The only information required to obtain the structure of the star and eigen-values of radial oscillation modes is the knowledge of the EOS. For a given EOS, equations (2)-(5) are solved numerically by standard techniques under the boundary conditions (11). While numerically integrating the equations for each EOS we make sure that the eigenfrequency of the fundamental mode goes to zero at the maximum mass of the star. We have also checked that the frequency also vanishes at the minimum stable mass too. For the EOS, as discussed in the introduction, we use two relativistic mean field theoretic models taken from Glendenning (Glendenning 1997) and two potential models incorporating relativistic corrections and three body interactions given by Akaml, Pandharipande and Ravenhall (Akaml 1998). Both class of models admit of mixed quark-nucleon phase in the core and correspond to matter as below :

RFT1H : pure confined hadronic phase for $nB < 0.26 fm^{-3}$, mixed confined phase at intermediate densities and pure quark-matter at $nB > 1.17 fm^{-3}$. Nuclear properties correspond to $K = 240$ MeV and $\frac{m^*}{m} = 0.78$.

RFT2H : confined hadronic phase for $nB < 0.26 fm^{-3}$, deconfined phase for $nB > 1.0 fm^{-3}$, mixed phase inbetween. $K = 300$ MeV and $\frac{m^*}{m} = 0.78$.

The models RFT1 and RFT2 correspond to pure hadronic phase occuring in the core of neutron stars without the existence of any quark phase.

PMT1H : Incorporating Argonne A18 potential along with three body interaction (A18+UIX model of Akaml et al. 1998) corresponds to pure confined hadronic phase for $nB < 0.1 fm^{-3}$, mixed phase at intermediate densities and pure phase for $nB > 0.49 fm^{-3}$.

PMT2H : Incorporating relativistic boost correction δv along with three body interaction UIX^* with Argonne A18 potential model corresponds to pure confined hadronic phase for $nB < 0.1 fm^{-3}$, and pure phase for $nB > 0.74 fm^{-3}$.

The models PMT1 and PMT2 refer to the above two models with pure confined hadronic phase.

The quark matter in the above EOS models is described by the Bag Model with $m_u = m_d = 0$, $m_s = 150$ MeV, the Bag constant $B^{\frac{1}{4}} = 180$ MeV and $\alpha_s = 0$.

To illustrate the effect of mixed phase on neutron star parameters, namely mass-radius relationship and on the frequency of radial oscillations, we have taken one model each from the above two categories considered. since the mass fraction contained in the crust of the star is a small fraction of the

total star mass ($< 2\%$), we have used the earlier results (Baym 1971; Lorenz 1993; Pethick 1995) of the EOS for matter densities $\leq 0.1 fm^{-3}$. In fig 1 we have plotted the M-R curves and find that the effect of the existence of mixed quark-nuclear matter phase in the core of neutron stars is to reduce the maximum mass. The effect is more pronounced for the relativistic mean field theory model than for the potential model. Whereas for the RFT models also radius corresponding to maximum mass decreases, for the potential models it increases instead. For smaller mass the two curves RFT2 and RFT2H (PMT2 and PMT2H) come very close to each other; this is to be expected since these points correspond to low central densities by which time the effect of deconfinement becomes negligible. In fig2 we have plotted the frequency of the fundamental mode and the next mode as a function of central density for the pure neutron and hybrid stars in the two categories and find that the frequency exhibits oscillatory behaviour in the case of a neutron star with a mixed quark-nuclear matter core. In tables (1a)-(4b), we provide the numerical results for the star parameters and radial mode frequencies of the models discussed in the text. In each table we list the central density ρ_c, radius R, mass M, red shift $Z = (1 - \frac{2GM}{R})^{-\frac{1}{2}} - 1$ of the stellar model and the frequencies of the first two radial models. The model above the stability limit is marked by a star.

4   Acknowledgements

TABLE 1a: RFT1

$\rho_c \times 10^{14}$ (gm/cc)	R (Km)	M ($M_\odot$)	Z	ν_0 (KHz)	nu_1 (KHz)
30.04	10.63	1.546	0.33	0.96	6.22*
25.35	10.95	1.550	0.31	0.45	6.32
15.22	12.09	1.496	0.257	1.52	6.26
10.65	12.78	1.394	0.22	1.89	6.16
8.79	13.07	1.314	0.19	2.09	6.14
7.00	13.35	1.187	0.17	2.38	6.05
5.83	13.48	1.072	0.14	2.65	6.26
4.70	13.58	0.868	0.11	2.81	5.67
3.61	13.70	0.608	0.07	2.89	4.01
3.07	13.86	0.471	0.05	3.04	3.18

TABLE 1b: RFT1H

$\rho_c \times 10^{14}$ (gm/cc)	R (Km)	M ($M_\odot$)	Z	ν_0 (KHz)	ν_1 (KHz)
31.68	10.20	1.45	0.31	0.89	6.30*
28.58	10.42	1.45	0.30	0.29	6.12
18.50	11.18	1.41	0.26	1.84	6.31
15.26	11.56	1.36	0.24	2.04	12.60
11.68	12.18	1.24	0.20	2.02	6.65
8.97	12.85	1.10	0.16	1.80	6.26
6.69	13.38	0.98	0.13	1.90	5.64
4.58	13.58	0.84	0.11	2.84	5.49
4.15	13.62	0.74	0.09	2.81	4.86
3.61	13.69	0.61	0.07	2.89	4.01

TABLE 2a: RFT2

$\rho_c \times 10^{14}$ gm/cc	R (Km)	M $(M_\odot)$	Z	ν_0 (KHz)	ν_1 (KHz)
30.82	10.88	1.645	0.346	1.38	5.976*
25.12	11.28	1.659	0.332	0.77	6.072*
23.50	11.42	1.661	0.326	0.50	6.076*
22.10	11.57	1.662	0.320	0.32	6.084
12.07	12.86	1.577	0.253	1.61	5.953
8.86	13.35	1.458	0.216	2.01	5.909
7.04	13.64	1.319	0.184	2.28	5.741
5.84	13.77	1.186	0.159	2.56	5.835
4.70	13.85	0.956	0.121	2.66	5.668
4.15	13.89	0.807	0.099	2.64	5.066
3.62	13.97	0.651	0.077	2.60	4.143
3.07	14.17	0.495	0.057	2.51	3.146
2.55	14.81	0.344	0.036	2.05	2.394

TABLE 2b: RFT2H

$\rho_c \times 10^{14}$ (gm/cc)	R (Km)	M $(M_\odot)$	Z	ν_0 (KHz)	ν_1 (KHz)
32.07	10.31	1.430	0.303	0.776	6.27*
27.40	10.68	1.434	0.289	0.213	6.13
20.88	11.40	1.425	0.261	0.851	5.57
17.39	11.68	1.409	0.248	1.426	5.83
13.27	12.20	1.346	0.219	1.723	6.16
10.92	12.66	1.268	0.193	1.721	6.14
8.60	13.22	1.160	0.163	1.641	5.72
6.48	13.68	1.041	0.136	1.795	5.09
5.26	13.83	0.958	0.122	2.262	5.00
4.10	13.89	0.807	0.099	2.643	5.07
3.62	13.97	0.651	0.077	2.603	4.14
3.07	14.17	0.495	0.056	2.515	3.15
2.55	14.81	0.344	0.036	2.054	2.40

TABLE 3a: PMT1

$\rho_c \times 10^{14}$ (gm/cc)	R (Km)	M ($M_\odot$)	Z	ν_0 (KHz)	ν_1 (KHz)
26.47	10.56	2.36	0.72	1.27	7.71*
24.33	10.70	2.37	0.70	0.60	7.86
20.45	11.02	2.35	0.65	1.53	8.14
17.05	11.36	2.30	0.58	2.31	8.38
14.08	11.69	2.19	0.48	2.92	8.53
11.47	11.98	1.96	0.39	3.40	8.51
16.34	12.17	1.61	0.28	3.70	8.21
9.17	12.20	1.38	0.23	3.73	7.91
8.11	12.21	1.13	0.17	3.67	7.39
7.10	12.23	0.87	0.13	3.50	6.52
5.21	12.35	0.65	0.09	3.29	5.26
4.31	12.80	0.45	0.06	2.82	3.48
3.42	13.41	0.35	0.04	2.41	2.65

TABLE 3b: PMT1H

$rho_c \times 10^{14}$ (gm/cc)	R (Km)	M ($M_\odot$)	Z	ν_0 (KHz)	ν_1 (KHz)
31.40	11.33	1.96	0.43	1.96	5.33*
20.59	11.49	1.97	0.43	0.46	6.67
13.31	11.89	1.89	0.38	2.51	7.61
11.29	12.05	1.80	0.34	3.05	7.70
9.07	12.15	1.65	0.29	3.54	7.95
8.95	12.18	1.56	0.27	3.71	8.17
7.41	12.21	1.33	0.22	3.73	7.81
7.30	12.22	1.18	0.18	3.49	7.10
6.33	12.23	0.92	0.13	3.54	6.73
5.95	12.26	0.82	0.12	3.46	6.31
5.39	12.32	0.70	0.10	3.36	5.61

TABLE 4a: PMT2

$\rho_c \times 10^{14}$ (gm/cc)	R (Km)	M ($M_\odot$	Z	ν_0 (KHz)	ν_1 (KHz)
27.10	10.03	2.188	0.68	0.64	8.49
17.49	10.82	2.066	0.52	2.90	8.98
13.47	11.22	1.831	0.39	3.55	8.93
12.28	11.34	1.711	0.35	3.71	8.81
11.14	11.43	1.569	0.30	3.82	8.64
10.05	11.51	1.403	0.25	3.88	8.38
9.01	11.57	1.220	0.21	3.88	8.01
8.01	11.63	1.025	0.16	3.79	7.51
5.19	12.17	0.520	0.07	3.11	4.40
3.42	14.20	0.269	0.03	1.80	2.22

TABLE 4b: PMT2H

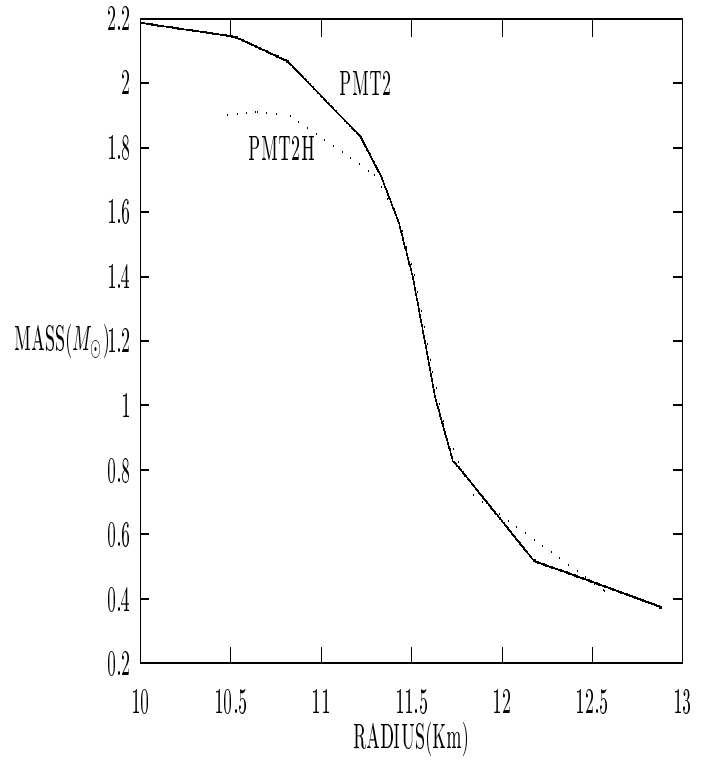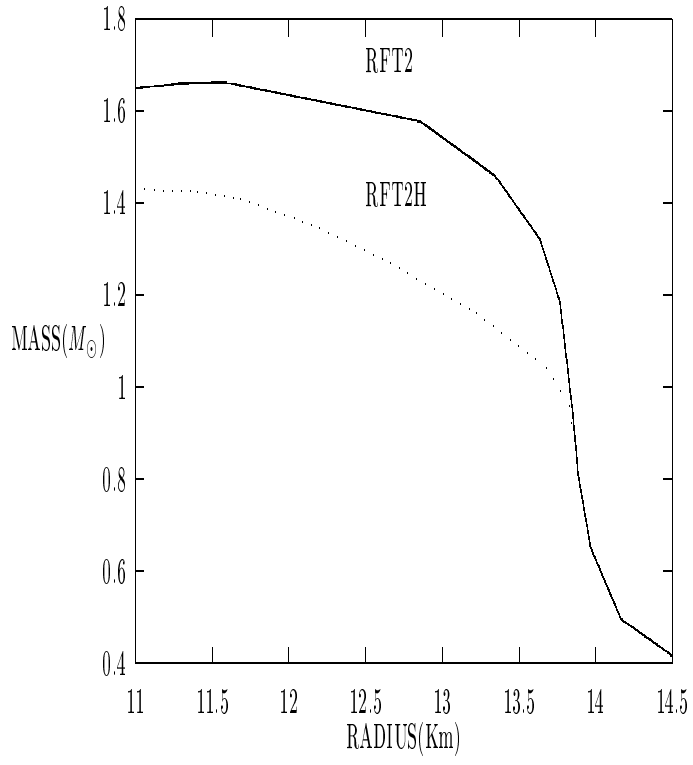| $\rho_c \times 10^{14}$ (gm/cc)	R (Km)	M ($M_\odot$)	Z	ν_0 (KHz)	ν_1 (KHz)
28.56	10.56	1.908	0.47	1.31	6.76*
24.64	10.65	1.911	0.46	0.50	7.25
20.04	10.83	1.898	0.44	1.86	7.83
17.13	11.00	1.861	0.42	2.48	8.15
14.97	11.16	1.801	0.38	2.93	8.30
13.24	11.29	1.716	0.35	3.30	8.34
11.78	11.40	1.606	0.31	3.62	8.37
11.12	11.45	1.542	0.29	3.76	8.43
10.49	11.49	1.471	0.27	3.86	8.48
9.42	11.55	1.295	0.22	3.89	8.18
8.40	11.62	1.104	0.18	3.84	7.74
7.42	11.70	0.908	0.14	3.70	7.10
6.48	11.84	0.721	0.10	3.48	6.14
5.56	12.04	0.589	0.08	3.25	5.05
4.65	12.57	0.426	0.05	2.84	3.48
3.77	13.68	0.305	0.35	2.14	2.45

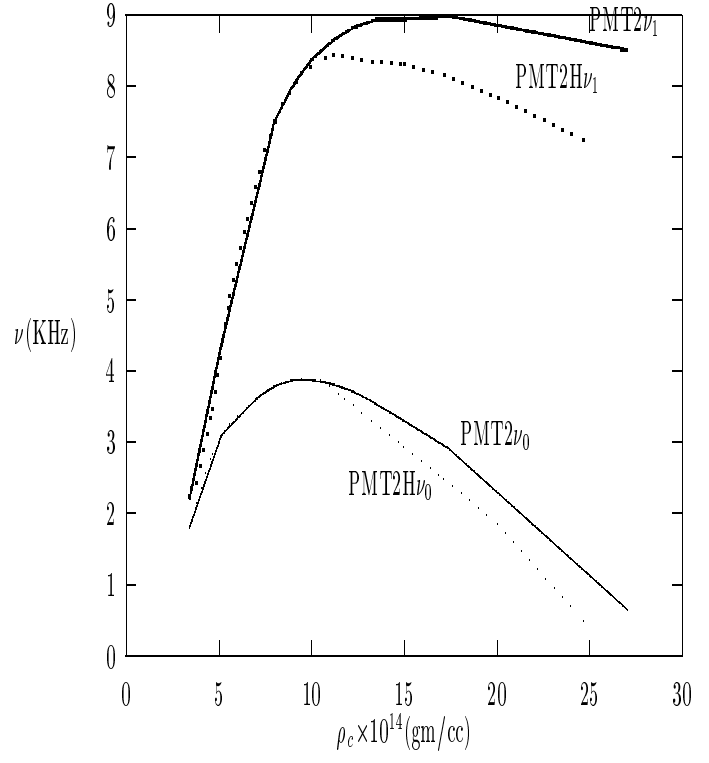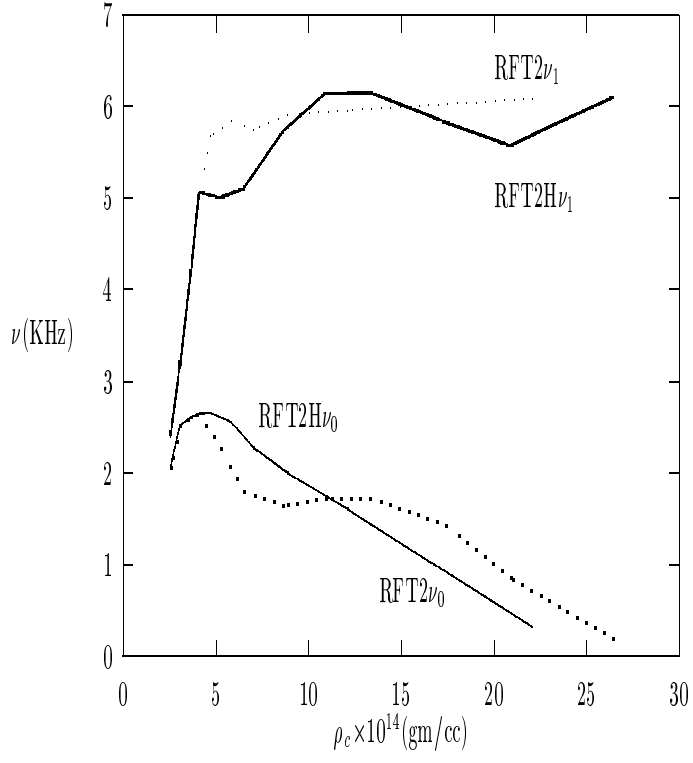Figure 1: *Plot of mass in solar mass unit Vs radius in Km.*

Figure 2: *Plot of frequency in KHz Vs central density in gm/cc*

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